Geometric representation theory and low-dimensional topology - workshop

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Abstracts

Aganagic, Mina

Knot categorification from geometry and mirror symmetry, via string theory

I will describe how two geometric approaches to categorification of RTW invariants of knots emerge from string theory. The first approach is based on a category of B-type branes on resolutions of slices in affine Grassmannians. The second is based on a category of A-branes in a Landau-Ginzburg theory. The relation between them is two dimensional (equivariant) mirror symmetry. String theory also predicts that a third approach to categorification, based on counting solutions to five dimensional Haydys-Witten equations, is equivalent to the first two. This talk is mostly based on joint work with Andrei Okounkov.

Berest, Yuri

Perverse sheaves, contact homology and cubical approximations

Knot contact homology is an interesting geometric invariant of a knot K in R3 defined by Floer-theoretic counting of pseudoholomorphic disks in the sphere conormal bundle of K in T R3. In its simplest form, this invariant was introduced by L. Ng and has been extensively studied in recent years by means of symplectic geometry and topology. In this talk, we will give a purely algebraic construction of knot contact homology based on the homotopy theory of (small) dg categories. For a link L in R3, we define a dg k-category AL with a distinguished object, whose quasi-equivalence class is a topological invariant of L. In the case when L is a knot, the endomorphism algebra of the distinguished object of AL coincides with a geometric dg algebra model of the knot contact homology of L constructed by Ekholm, Etnyre, Ng and Sullivan (2013). The input of our construction is a natural action of the Artin braid group Bn on the category of perverse sheaves on a two-dimensional disk with singularities at n marked points, studied by Gelfand, MacPherson and Vilonen (1996). Time permitting, we will also discuss a possible generalization of contact homology to arbitrary spaces using the homotopy theory of cubical diagrams of simplicial sets.

Brochier, Adrien

Quantization of character varieties, topological field theories and the Riemann-Hilbert correspondence

I will give a gentle exposition of some recent work joint with D. Ben-Zvi, D. Jordan and N. Snyder on quantization of character varieties of surfaces, using the language of topological field theories. I will first recall various points of view on the representation category of quantum groups and the associated topological invariants. I will then explain how, using the formalism of factorization homology, one can construct and compute explicitly a certain category-valued invariant of surfaces. This provides a canonical quantization of the Atiyah--Bott-Goldman Poisson structure on character varieties, unifies and generalizes a number of previously known constructions and has interesting applications in representation theory and low-dimensional topology. Time permitting, I will mention a work in progress on a higher genus analog of the Kohno-Drinfeld theorem, using this formalism to compute explicitly the monodromy of certain differential equations arising in conformal field theory. This can be thought of as a quantization of the Riemann-Hilbert correspondence.

Cherednik, Ivan

DAHA superpolynomials for iterated links

It will be mostly an introduction to our long paper with Ivan Danilenko on DAHA superpolynomials of (colored) torus iterated links. Connections with the HOMFLY-PT polynomials and the Khovanov-Rozansky stable reduced polynomials will be briefly discussed. The theory of the latter is mostly developed for uncolored knots, especially in the reduced setting. Also, DAHA superpolynomials satisfy the super-duality (a theorem) and have other remarkable symmetries, which are generally difficult to approach topologically. Though the most clarifying proof of the super-duality for algebraic links is via the so-called motivic superpolynomials of plane curve singularities, conjecturally coinciding with the DAHA ones. The super-duality becomes then the functional equation, a fundamental development, which may have physics implications. The motivic direction inspired the Riemann hypothesis for DAHA superpolynomials (to be touched a bit at the end), but it will be omitted in this talk. We will focus on the DAHA theory of colored iterated links, explaining the main steps and providing some examples.

Dimofte, Tudor

3d mirror symmetry and HOMFLY-PT homology

Recently, Gorsky-Negut-Rasmussen and Oblomkov-Rozansky proposed constructions of HOMFLY-PT link homology related to coherent sheaves on Hilbert schemes. Oblomkov-Rozansky explained how their construction was realized in several different physical systems, including (most relevantly for this talk) 3d supersymmetric gauge theories. I will discuss how a duality of 3d gauge theories known as 3d mirror symmetry --- whose mathematical/categorical implications are only starting to be explored --- acts on the setups above. The result is a very different, "A type" construction of link homology, related to cohomology of affine Springer fibers and to Hilbert schemes of points on singular curves. The talk will attempt to be fairly broad, with no prior technical knowledge assumed. Based on work with N. Garner, J. Hilburn, A. Oblomkov, and L. Rozansky; and related work of N. Garner and O. Kivinen.

Etingof, Pavel

Short star-products for filtered quantizations

{\bf Abstract:} Let A be a filtered Poisson algebra with Poisson bracket $\lbrace{,\rbrace}$ of degree -2. A {\it star product} on A is an associative product $*: A\otimes A\to A$ given by $$a*b=ab+\sum_{i\ge 1}C_i(a,b),$$ where C_i has degree $-2i$ and $C_1(a,b)-C_1(b,a)=\lbrace{a,b\rbrace}$. We call the product * {\it even} if $C_i(a,b)=(-1)^iC_i(b,a)$ for all i, and call it {\it short} if $C_i(a,b)=0$ whenever $i>{\rm min}({\rm deg}(a), {\rm deg}(b))$.
Motivated by three-dimensional $N=4$ superconformal field theory, In 2016 Beem, Peelaers and Rastelli considered short even star-products for homogeneous symplectic singularities (more precisely, hyperK\""ahler cones) and conjectured that that they exist and depend on finitely many parameters. We prove the dependence on finitely many parameters in general and existence for a large class of examples, using the connection of this problem with zeroth Hochschild homology of quantizations suggested by Kontsevich. Beem, Peelaers and Rastelli also computed the first few terms of short quantizations for Kleinian singularities of type A, which were later computed to all orders by Dedushenko, Pufu and Yacoby. We will discuss some generalizations of these results. This is joint work with Eric Rains and Douglas Stryker.

Francis, John

Moduli of stratifications and factorization homology

The Ran space Ran(X) is the space of finite subsets of X, topologized so that points can collide. Ran spaces have been studied in diverse works from Borsuk–Ulam and Bott, to Beilinson–Drinfeld, Gaitsgory–Lurie and others. The alpha form of factorization homology takes as input a manifold or variety X together with a suitable algebraic coefficient system A, and it outputs the sheaf homology of Ran(X) with coefficients defined by A. Factorization homology simultaneously generalizes singular homology, Hochschild homology, and conformal blocks or observables in conformal field theory. This alpha form of factorization homology has applications to the study of mapping spaces in algebraic topology, bundles on algebraic curves, and perturbative quantum field theory. There is also a beta form of factorization homology, where one replaces the Ran space with a moduli space of stratifications, designed to overcome certain strict limitations of the alpha form. The key notion is that of a constructible bundle: in terms of its functor of points, a K-point of this moduli space is a constructible bundle over K. One application is to proving the Cobordism Hypothesis, after Baez–Dolan, Costello, Hopkins–Lurie, and Lurie. This is joint work with David Ayala.

Gunningham, Sam

q-Character sheaves and Springer theory

The category of equivariant $D_q(G)$-modules sits at the interface of low-dimensional topology and geometric representation theory. It appears naturally in the context of factorization homology and skein theory, but it may also be thought of as a q-deformation of the category of equivariant modules for the ring D(G) of differential operators on a complex reductive group G. In this talk I will survey various aspects of an ongoing program to adapt the powerful tools available in the D-module setting (e.g. character sheaves, parabolic induction, Springer theory) to the realm of D_q-modules. As an example, I will present a new derivation of a recent result of Carrega and Gilmer on the 9-dimensionality of the skein module for a 3-torus. This is joint work with (combinations of) David Jordan, Monica Vazirani, and Pavel Safronov.

Gukov, Sergei

New TQFTs from DAHA

To follow

Haiden, Fabian

Legendrian skein algebras and Hall algebras

This is a report on work in progress in understanding the relation between skein algebras and Hall algebras of Fukaya categories, motivated by a recent paper of Cooper-Samuelson. Another motivation is the problem of defining Hall algebras for Z/2-graded categories, and the limiting case q=1. The main novelty is to assign to a Lagrangian submanifold a positive linear combination of isomorphism classes of objects in the Fukaya category (i.e. a “random object”).

Mellit, Anton

$A_{q,t}$ algebra as a partially symmetrized DAHA and computations of knot invariants

$A_{q,t}$ algebra arose from my work with Erik Carlsson on the shuffle conjecture. I will explain how this algebra can be constructed naturally from a topological (skein-theoretic) point of view. Similarly to the way representation theory of quantum groups gl_N can be interpolated in the limit $N\to\infty$, the $A_{q,t}$ algebra arises if one attempts to interpolate the symmetric power of the standard representation Sym_M in the limit $M\to\infty$. Then I will show how the algebra was used to compute the triply graded homology (superpolynomials) of torus knots.

Negut, Andrei

The PBW basis of the quantum toroidal algebra

We give a presentation of $U_{q,q'}(gl_n^{\wedge\wedge})$ via generators and relations, akin to the one developed by Burban-Schiffmann in the elliptic Hall algebra (which would correspond to n=1). One hopes that our presentation could help define a version of the elliptic Hall algebra for all n, although this is still only a dream.

Oblomkov, Alexei

Dualisable link homology and 3D TQFT

Talk is based on the joint work with Lev Rozansky. It is elementary to see that the HOMFLYPT polynomial of a knot K $P_K(a,q)$ is palindromic: $P_K(a,q)=P_K(a,1/q)$. It was conjectured by Dunfield-Gukov-Rasmussen that the Poincare polynomial $\mathcal{P}_L(a,q,t)$ of the HOMFLYPT homology has similar property. In my talk I explain a proof of the conjecture that relies on the construction 3D TQFT that provides a mathematical model for 3D sigma model of Kapustin-Rozansky-Saulina. As by-product of our construction we find an interpretation of the knot homology as v space of sections of a particular sheaf on the Hilbert scheme of points on the plane as well as another explanation why the skein algebra of torus is an elliptic Hall algebra.

Stroppel, Catharina

DAHA actions on fusion rings

In this talk I will describe fusion algebras arising from quantum groups at roots of unity. After a short overview of the general theory we study the rings in more details. The goal is to construct actions of certain double affine Hecke algebras on these algebras.

Pei, Du

New TQFTs from DAHA (Part 2)

We will continue to study DAHA and its representations via quantum physics, focusing on new connections with mirror symmetry and skein theory.

Safronov, Pavel

R-matrices with a spectral parameter via shifted Poisson structures

Constant (dynamical) R-matrices have a categorical interpretation in terms of braided monoidal categories equipped with a monoidal forgetful functor to the category of vector spaces (Harish—Chandra bimodules). In the first part of the talk I will review analogous interpretations of classical r-matrices in terms of shifted Poisson structures. In the second part I will describe a conjectural interpretation of classical (dynamical) r-matrices with a spectral parameter (describing Yangians, quantum affine algebras and elliptic quantum groups) in terms of shifted Poisson structures and their quantizations.

Scheimbauer, Claudia

En-algebras, extended topological field theories and dualizability

The Cobordism Hypothesis provides a beautiful interplay between extended topological field theories and dualizability conditions, allowing for a conceptual explanation of certain finiteness conditions appearing in representation theory, and, vice versa, a geometric understanding of algebraic objects. In turn, En-algebras, which are algebras for the little disks operad, (e.g. associative algebras and quantum groups) provide important examples. I will explain why every En-algebra leads to a categorified topological field theory using dualizability arguments. Furthermore, I will explore extensions and further directions.

Snyder, Noah

Local topological field theories with values in Morita categories

The ordinary Morita symmetric monoidal 2-category has objects that are algebras over a field k, 1-morphisms from A to B are A-B bimodules, and 2-morphisms are bimodule maps, and the monoidal structure is giving by tensoring over k. More generally one can consider an Morita (r+m)-category whose objects are E_r-algebras in some m-category, 1-morphisms are E_{r-1}-bimodules, etc. Some notable specific examples are the Morita 3-category of monoidal categories and the Morita 4-category of braided monoidal categories. An n-dimensional local topological field theory is a symmetric monoidal functor from a bordism n-category to some target n-category. The Hopkins-Lurie-Baez-Dolan cobordism hypothesis says that such TFTs are classified in terms of n-dualizable objects in the target. The goal of this talk is to give a survey of what is known about n-dualizability of objects in Morita categories. This will touch on results of Calaque-Scheimbauer, Haugseng, Johnson--Freyd-Scheimbauer, Gwilliam-Scheimbauer, Brandenburg-Chirvasitu-Johnson--Freyd, and Morrison-Walker, and focus on my joint work with Douglas-Schommer--Pries and Brochier-Jordan on dualizability of tensor categories and braided tensor categories.

Teleman, Constantin
Coulomb branches, passée topology and non-polarizable matter
To follow

Williams, Harold
Canonical bases for Coulomb branches
Following work of Kapustin-Saulina and Gaiotto-Moore-Neitzke it is anticipated that the expectation values of irreducible half-BPS line defects define a canonical basis in the quantized Coulomb branch of a 4d N=2 field theory. In this talk we propose a mathematical definition of the category of such defects, hence of the associated canonical basis, in the case of N=2 gauge theories of cotangent type. The form of the definition is a finite length t-structure on the DG category of coherent sheaves on a derived enhancement of the space of triples introduced by Braverman-Finkelberg-Nakajima. This heart of this t-structure is a non-Noetherian extension of the category of perverse coherent sheaves on the affine Grassmannian, to which it specializes in the case of pure N=2 gauge theory. It is expected that these categories categorify the cluster algebra associated to the BPS quiver of the theory, which we confirm in various examples. This is work in progress with Sabin Cautis.

Wyss, Dimitri
Non-archimedean integrals on the Hitchin fibration
Non-archimedean or p-adic integration is an analytic tool to study rational points of algebraic varieties over finite fields. Dener-Loser and Batyrev have realized that this can be used in some cases to study the topology of complex algebraic varieties. We apply this idea to the moduli spaces of G-Higgs bundles M(G) and show in particular, that for a pair of Langlands dual groups the corresponding moduli spaces have the same non-archimedean volume. As a geometric application we find an agreement of (stringy) Hodge numbers of M(SLn) and M(PGLn) as predicted by a conjecture of Hausel-Thaddeus. For general G this leads to a new proof of the geometric stabilization theorem, a key ingredient in Ngô's proof of the fundamental lemma. This is joint work with Michael Groechenig and Paul Ziegler.